Let $\vec{l}$ be the vector $5\vec{i} - 7\vec{j}$.

Let $\vec{g}$ be the vector $< -12, -4 >$.

SCORE: _____ / 45 PTS

[a] Find a vector of magnitude 8 in the opposite direction as $\vec{g}$. Write your final answer as a linear combination of $\vec{i}$ and $\vec{j}$.

$$-8\left(\frac{1}{\|\vec{g}\|}\right)\vec{g} = \frac{-8}{\sqrt{(-12)^2+(-4)^2}}\langle -12,-4\rangle = \frac{-8}{4\sqrt{10}}\langle -12,-4\rangle = -\frac{\sqrt{10}}{5}\langle -12,-4\rangle$$

$$= \frac{12\sqrt{10}}{5}\vec{i} + \frac{4\sqrt{10}}{5}\vec{j}$$

$$\approx 7.6\vec{i} + 2.5\vec{j}$$

[b] If $\vec{l}$ is perpendicular to $\vec{r} = a\vec{i} + (a+4)\vec{j}$, find the value of a.

$$\vec{l} \cdot \vec{r} = 0 \longrightarrow 5a - 7(a+4) = 0$$

$$-2a - 28 = 0$$

$$a = -14$$

[c] Find the projection of $-10\vec{j}$ onto $\vec{l}$.

$$\frac{-10\vec{j}\cdot\vec{l}}{\vec{l}\cdot\vec{l}}\vec{l} = \frac{70}{74}\langle 5,-7\rangle = \left\langle \frac{350}{74}, \frac{-490}{74}\right\rangle \approx \langle 4.7, -6.6\rangle$$

[d] Find the angle between $\vec{l}$ and $\vec{g}$.

$$\cos^{-1}\frac{\vec{l}\cdot\vec{g}}{\|\vec{l}\|\|\vec{g}\|} = \cos^{-1}\frac{-32}{(\sqrt{74})(4\sqrt{10})} \approx 111.7°$$

[e] If $\vec{l}$ represents a force vector that is applied to an object to move it from the point $(1, -3)$ to the point $(-2, -9)$, find the work done.

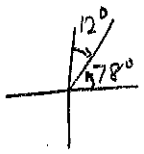
$$\vec{d} = \langle -2-1, -9-(-3)\rangle = \langle -3, -6\rangle$$

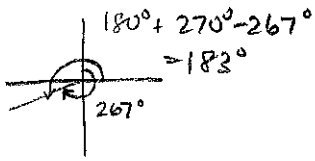
$$\vec{l}\cdot\vec{d} = 27$$

A boat sails 21 nautical miles on a bearing of 12° from port,
then changes direction, and sails an additional 13 nautical miles on a bearing of 267°.

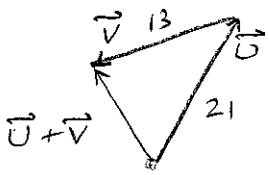
SCORE: _____ / **20 PTS**

[a] Find the vector which represents the boat's final position relative to port.





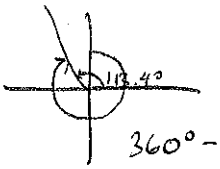
$$\vec{u}+\vec{v} = 21\langle \cos 78^\circ, \sin 78^\circ\rangle + 13\langle \cos 183^\circ, \sin 183^\circ\rangle$$
$$\approx \langle -8.6, 19.9\rangle$$



[b] What is the bearing of the boat's final position relative to port ?

$$\theta_{\vec{u}+\vec{v}} = 180^\circ + \tan^{-1}\frac{19.9}{-8.6} \approx 113.4^\circ$$

BEARING = 336.6°



$360^\circ - 113.4^\circ + 90^\circ = 336.6^\circ$

[c] How far is the boat from port at the end ?

$$\|\vec{u}+\vec{v}\| = \sqrt{(-8.6)^2 + 19.9^2} \approx 21.7 \text{ NAUTICAL MILES}$$

Suppose that $S = 72^\circ$ and $p = 28$. Find all values of s for which there are two possible triangles ΔSPA.

SCORE: _____ / **5 PTS**

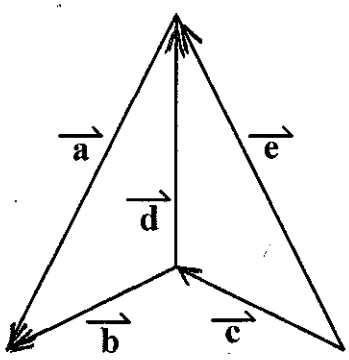
$$p \sin S < s < p$$
$$28 \sin 72^\circ < s < 28$$
$$26.6 < s < 28$$

MULTIPLE CHOICE: **Circle the letter corresponding to the correct answer.** **SCORE:** _____ / 5 PTS

If $d = 3.2$ and $n = 1.9$, there is a possible triangle ΔDNE if $e =$ $3.2+1.9>5.0$

[A] 1.1 [B] 5.2 [C] 1.3 [D] 5.0 [E] none of the above

Write vectors $\vec{d}$ and $\vec{e}$ in terms of vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ in the diagram below. **SCORE:** _____ / 5 PTS



$\vec{d}+\vec{a}=\vec{b} \rightarrow \vec{d}=\vec{b}-\vec{a}$

$\vec{c}+\vec{d}=\vec{e} \rightarrow \vec{e}=\vec{c}+\vec{b}-\vec{a}$

Solve triangle ΔFUN if $U = 64°$, $\not{u} = 7.8$ and $f = 8.3$. **Sketch and label triangles with your final answers.** **SCORE:** _____ / 15 PTS
If no such triangle exists, write "DNE". If more than one triangle is possible, solve for all possible triangles.

$f \sin U = 8.3 \sin 64° \approx 7.5$

$f \sin U < u < f \rightarrow$ 2 Δ's

$\frac{\sin F}{8.3} = \frac{\sin 64°}{7.8}$

$\sin F = \frac{8.3 \sin 64°}{7.8}$

$F \approx 73.0°$ OR $180° - 73.0° = 107.0°$

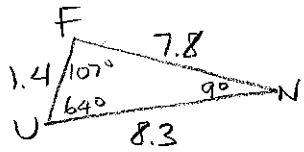
$N = 180° - (64° + 73°)$

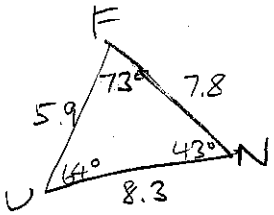
$= 43°$

$\frac{n}{\sin 43°} = \frac{7.8}{\sin 64°} \rightarrow n = \frac{7.8 \sin 43°}{\sin 64°} \approx 5.9$

$N = 180° - (64° + 107°) = 9°$

$\frac{n}{\sin 9°} = \frac{7.8}{\sin 64°} \rightarrow n = \frac{7.8 \sin 9°}{\sin 64°} = 1.4$





Find the area of triangle ΔQED if $q = 4.1$, $d = 10.5$, $Q = 21°$, $E = 46°$ and $D = 113°$. **SCORE:** _____ / 5 PTS

$\frac{1}{2}(4.1)(10.5)\sin 46° \approx 15.5$